ON THE SELF-MODULATION IN THE BIOLOGICAL STRUCTURE

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Abstract. In the dynamics analysis based on the electromagnetic biological fields propagation, the conservation of the surface element from phase space implies special temporal stochasticities. Then, the identification of temporal stochasticities in the phase space, meant to transform dissipative dynamics into non- dissipative dynamics, implies Riccati-type gauge at various scale resolutions, *i.e.* self-modulation of the Stoler type.

Keywords: biostructure; operational procedure; Riccati gauge; self-modulation.

1. Introduction

The electromagnetic biological field is vital in bio-structure dynamics. No matter the type of bio-structure (bacteria, virus, etc.), they are "specialized"

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both in "generating" and "manipulating" biological currents: *e.g.* myocardial cells generate biological currents which allow the cardiac muscle to contract at the right time, any nervous system specific to any bio-structure uses biological currents in order to transmit signals along the entire bio-structure, making thus possible its dynamics.

Moreover, the majority of bio-structure is polarizable and excitable, so that "action potentials" can be associated with their dynamics ("elementary bioelectric events" which are "sources" for all the bio-electromagnetic signals in bio-structures) (Grimnes and Martinsen, 2015; Malmivuo and Plonsey, 1995).

In the present paper we propose a theoretical model for analyzing biostructures dynamics through various operational procedures (phase space analysis, stochastic analysis, etc.).

2. Mathematical Model

Let us admit that bio-structures are complex systems (Badii and Politi, 1997; Mitchell, 2009) associated with fractals (Mandelbrot, 1983; Feder and Aharoner, 1990) and the electromagnetic biological field is responsible for bio-structures dynamics through Maxwell-type equations (Grimnes and Martinsen, 2015; Malmivuo and Plonsey, 1995). The type of bio-structure is introduced by means of electrical permittivity ε, magnetic permeability μ, and the electrical conductivity σ, all of these being dependent on scale resolution (Feder and Aharoner, 1990; Nottale, 2011). In such a context, the behavior of bio-structures is controlled both by biological currents and by the electrical and magnetic components of the biological electromagnetic field. Because we propose to analyze only temporal dynamics of bio-structures, such behaviors are contained in the differential equation:

$$\mu\varepsilon \frac{\partial^2 T(t)}{\partial t^2} + \mu\sigma \frac{\partial T(t)}{\partial t} + \alpha^2 T(t) = 0 \qquad (1)$$

where $T(t)$ can be the temporal components either of the biological current or of the biological electromagnetic field, and $\alpha > 0$ is a variables separation constant which will be defined later on.

Let us analyze Eq. (1) in the phase space, by means of substitutions:

$$Q(t) = T(t), \; A = \mu\varepsilon, \; 2B = \mu\sigma, \; C = \alpha^2 \qquad (2)$$

Then, Eq. (1) becomes:

$$A\ddot{Q} + 2B\dot{Q} + CQ = 0 \qquad (3)$$

where Q is a variable with which we can describe the bio-structures dynamics, and A, B, C are real positive coefficients depend on scale resolution. The same equation can be written as a system:

$$\dot{P} = -\frac{2B}{A}P - \frac{C}{A}Q, \qquad \dot{Q} = P \tag{4}$$

The second equation from (4) can be a relevant moment-type variable, if and only if Q is a relevant coordinate-type variable for bio-structures dynamics in a phase space (P, Q). Accepting such a "theoretical construct", the system (4) is not a Hamiltonian system, as would be expected when discussing about relevant coordinate and moment, because its matrix is not an involution (it does not have a null trace), (Arnold, 1989). This fact becomes obvious if the system is written in the form:

$$\begin{pmatrix} \dot{P} \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} -\dfrac{2B}{A} & -\dfrac{C}{A} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} \tag{5}$$

As long as the physical parameters of the 2×2 matrix, *i.e. A, B, C*, are constant, the system can be rewritten in an equivalent form which highlights the energy status, and thus, the Hamiltonian status. Indeed, from Eq. (5) we can easily obtain the differential equation:

$$\frac{A}{2}\left(P\dot{Q} - Q\dot{P}\right) = -\frac{1}{2}(AP^2 + 2BPQ + CQ^2) \tag{6}$$

which proves that the energy, *i.e.* the quadratic form from the right side of Eq. (6), is the variation rate of the physical action represented by the elementary surface from the (P, Q) phase space (Arnold, 1989). Eq. (6) is a Riccati-type equation for a certain variable, be it $w = P/Q$, so that it can be written as:

$$A\dot{w} + Aw^2 + 2Bw + C = 0 \tag{7}$$

It is not necessary to find a direct physical meaning for *w*, at least for now. We must note though that the solution of Eq. (7) is given by the ratio of the Hamiltonian system's (5) solutions, *i.e.*

$$\begin{pmatrix} \dot{P} \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} -\dfrac{B}{A} & -\dfrac{C}{A} \\ 1 & \dfrac{B}{A} \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix} \tag{8}$$

This is a generic trait of relations between Riccati's equations and Hamiltonian dynamics (Zelkin, 2001). We can go back to Eq. (6) and introduce the 1-form differential from (8) which characterizes the elementary surface from the phase space. Regarding Eq. (7), it can be integrated in order to specify that the energy is no longer conserved, and a conservation law exists for the quantity (Denman, 1968):

$$F(P,Q) =$$
$$= \frac{1}{2}(AP^2 + 2BPQ + CQ^2)\, exp\left\{\frac{2B}{\sqrt{AC - B^2}}\, \tan^{-1}\left(\frac{AP + BQ}{Q\sqrt{AC - B^2}}\right)\right\} = \quad (9)$$
$$= const.$$

It can be observed that the energy is conserved, in a classical sense, if only if either the B coefficient is null, or the phase plane motion takes place along a line passing through the origin, with its slope determined by the B/A ratio. Written as:

$$\frac{CQ^2}{2} = \frac{const.}{(1 + 2rw + w^2)}\, exp\left\{2\frac{r}{\sqrt{1 - r^2}}\tan^{-1}\left(\frac{w\sqrt{1 - r^2}}{1 - rw}\right)\right\} \quad (10)$$

with notations:
$$w^2 = \frac{AP^2}{CQ^2}, \qquad r^2 = \frac{B}{AC} \quad (11)$$

Eq. (9) shows a striking resemblance with the distribution function over a statistical ensemble of local oscillators, given by relation (Lavenda, 1992):

$$P(w,r) = exp(-\beta\varepsilon_0) =$$
$$= \frac{1}{(1 + 2rw + w^2)}\, exp\left\{2\frac{r}{\sqrt{1 - r^2}}\, \tan^{-1}\left(\frac{w\sqrt{1 - r^2}}{1 - rw}\right)\right\} \quad (12)$$

with
$$w = \frac{\varepsilon_0}{u} \quad (13)$$

ε_0 the energy quantum, u is the reference energy, r the correlation coefficient and β is the Boltzmann's factor. Thus, it results that the potential energy effectively depends on the ratio between the kinetic energy and the potential energy and that this ratio is a statistical variable through which the two energies can be determined.

The present theory highlights one of the most interesting properties of energy, in relation to the conservation law. The standard equation:

$$\ddot{Q} + 2w_0^2 Q = 0 \tag{14}$$

with

$$w_0^2 = \frac{C}{A} \tag{15}$$

is a consequence of the stationary property of the mean of the difference between the kinetic energy and the potential energy along the entire bio-structure dynamics. As known, the difference between the kinetic energy and the potential energy defines the dynamics Lagrangean, L,

$$L = \frac{AP^2}{2} - \frac{CQ^2}{2} \tag{16}$$

The integral of this difference between two time moments, t_1 and t_2, defines the action S:

$$S(t_1, t_2) = \int_{t_1}^{t_2} \left(\frac{AP^2}{2} - \frac{CQ^2}{2} \right) dt \tag{17}$$

which is thus proportional with the mean of the difference between the kinetic and the potential energy, for the bio-structure dynamic during the considered time interval. It can be thus stated that the bio-structure dynamics, described through Eq. (14), distribute the two types of energy in such away in which their mean along the entire duration of the dynamics is stationary.

We cannot exactly apply the same reasoning for obtaining Eq. (3), but the above considerations leads us to statistical arguments which are not modified *per se*. Thus, for Eq. (3) it is not the time mean of the difference between the kinetic and potential energies which changes, but the time mean of the function:

$$L(Q, \dot{Q}, t) = \frac{1}{2} \left(A\dot{Q}^2 - CQ^2 \right) exp\left(\frac{2B}{A} t \right) \tag{18}$$

The action corresponding to such a situation takes place on a finite time interval given by the integral:

$$S(t_1, t_2) = \int_{t_1}^{t_2} \left(A\dot{Q}^2 - CQ^2 \right) exp\left(\frac{2B}{A} t \right) dt \tag{19}$$

Action (19) can be interpreted as the time mean of the difference between the kinetic energy and potential energies, for the *a priori* statistical measure of the time domain defined by an exponential distribution. In the case of Eq. (19) this distribution is uniform. Thus, it can be stated that, through the Hamiltonian theory of motion, the motion equations, given either by Eq. (3) or Eq. (14), are clear consequences of a temporal statistic. They are different only by the *a priori* mean of the time axis specific to the described dynamics. It is therefore to be expected that, in the more general case, the temporal statistic itself to change.

But what would be the reason behind changing the temporal statistic? This becomes obvious if we take into account that the Lagrangean (18) has parameters *A* and *C* exponentially variable:

$$A(t) \rightarrow A \, exp\left(\frac{2B}{A}t\right), \qquad C(t) \rightarrow C \, exp\left(\frac{2B}{A}t\right) \tag{20}$$

Therefore, the sum of the temporal statistic is given by the variability of the parameters *A* and *C*, which specifies that, from a physical point of view, the analyzed bio-structure is never isolated, but in a permanent interaction with the biological medium.

The situation described by Lagrangean (18) is a totally particular case of a much more general situation, which imposed the idea of gauge (Riccati gauge).

For this sense let us define a gauge in which Lagrangean (18) is a perfect square. This procedure is well known and frequently used in control theory (Zelkin, 2000). Now, the cyclicitycondition, becomes essential. It all resumes to adding to Lagrangean (18) the term:

$$\frac{1}{2}\frac{d}{dt}\left[w \, exp\left(\frac{2B}{A}t\right)Q^2\right] \tag{21}$$

where *w* is a continuous time function, requiring that the Lagrangean be a perfect square. The time interval variation of the function under the new derivation operator is null because of a special restriction (this bio-structure dynamics "begin" and "end" in the same point), therefore the dynamic Eq. (3) is not changed.

The new Lagrangean, in $\dot{Q}$ and Q coordinates, becomes

$$L\left(Q, \dot{Q}, t\right) = \frac{1}{2}A \, exp\left(\frac{2B}{A}t\right)\left(\dot{Q} + \frac{w}{A}Q\right)^2 \tag{22}$$

with the condition that *w* satisfied the Riccati-type differential equation:

$$A\dot{w} - w^2 + 2Bw + AC = 0 \qquad (23)$$

With condition (23), Lagrangean (22) leads us to the same Eq. (3) as Lagrangean (18). As before, there is a connection between the Riccati –type Eq. (23) and the Hamiltonian dynamics. We find in this situation that the matrix system has the expression:

$$\begin{pmatrix} \dot{\eta} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} -\dfrac{B}{A} & \dfrac{C}{A} \\ -1 & \dfrac{B}{A} \end{pmatrix} \begin{pmatrix} \eta \\ \xi \end{pmatrix} \qquad (24)$$

with

$$w = \frac{\eta}{\xi} \qquad (25)$$

which rigorously represents a Hamiltonian system. Therefore, variables η and ξ must be identified with coordinates from the phase space.

With regard to the solution of the Riccati-type Eq. (23), we must first notice that the roots of the polynomial

$$P(w) = w^2 - 2Bw - AC \qquad (26)$$

can be written as:

$$w_1 \equiv B + iA\Omega, \qquad w_2 \equiv B - iA\Omega, \quad w^2 = \frac{C}{A} - \left(\frac{B}{A}\right)^2 \qquad (27)$$

Performing the homographic transformation

$$z = \frac{w - w_1}{w - w_2} \qquad (28)$$

it results through direct calculus that z is a solution of the linear and homogenous first order equation

$$\dot{z} = 2i\omega z \qquad (29)$$

which allows the solution

$$z(t) = z(0)e^{2i\omega t} \qquad (30)$$

Therefore, if the initial condition $z(0)$ is conveniently expressed, the general solution of Eq. (23) can be found, by writing transformation (28) as:

$$w = \frac{w_1 + re^{2i\omega(t-t_r)}}{1 + re^{2i\omega(t-t_r)}} \tag{31}$$

where r and t_r are two real constants which characterize the solution. By using (27) we can write this solution in real terms, as

$$z = B + A\omega\left(\frac{2r\sin[2\omega(t-t_r)]}{1 + r^2\cos[2\omega(t-t_r)]} + i\frac{1-r^2}{1 + r^2\cos[2\omega(t-t_r)]}\right) \tag{32}$$

which highlights a self-modulation of the pulsation-type characteristic ω known as the Stoler transformation (Stoler, 1970; Stoler, 1971), implying a complex form for this parameter. In Fig. 1 we present this self-modulation phenomenon through Re(z) time dependences, for various values of r and ω.

The dependences of Re(z) on r and ω (3D and contour dependences) at various scale resolutions are shown in Fig. 2.
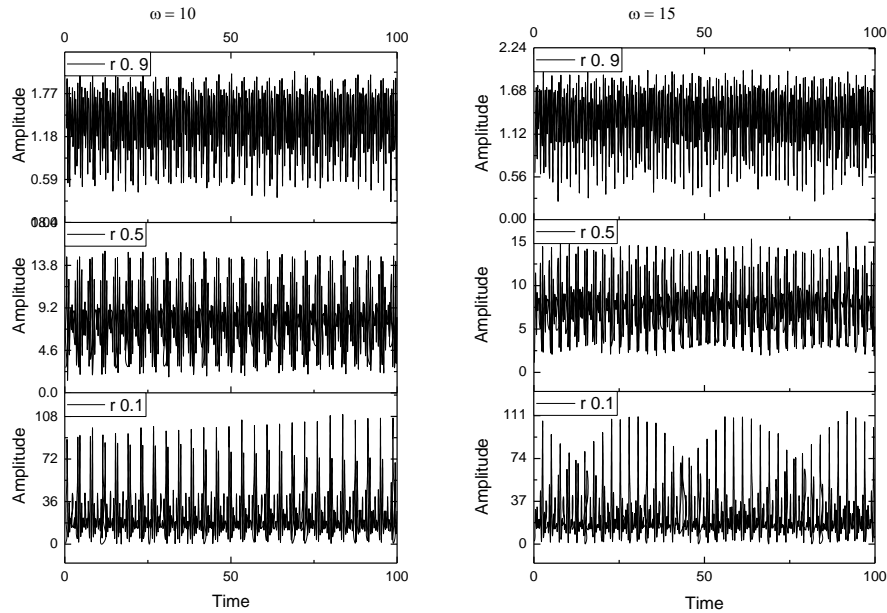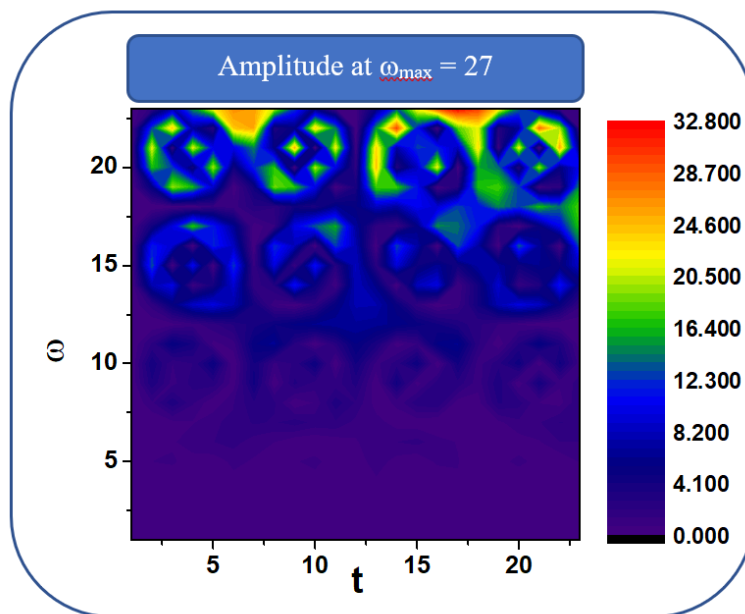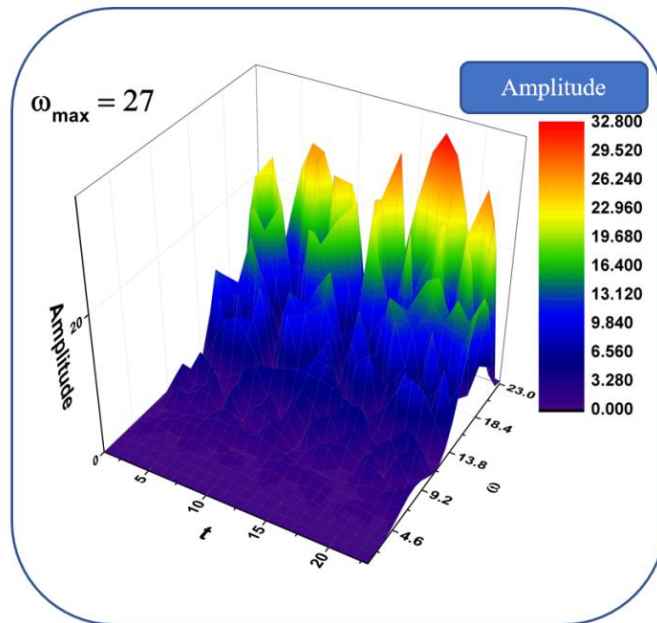

Fig. 1 – Amplitude variation with time of the Re(z) solution
for four different
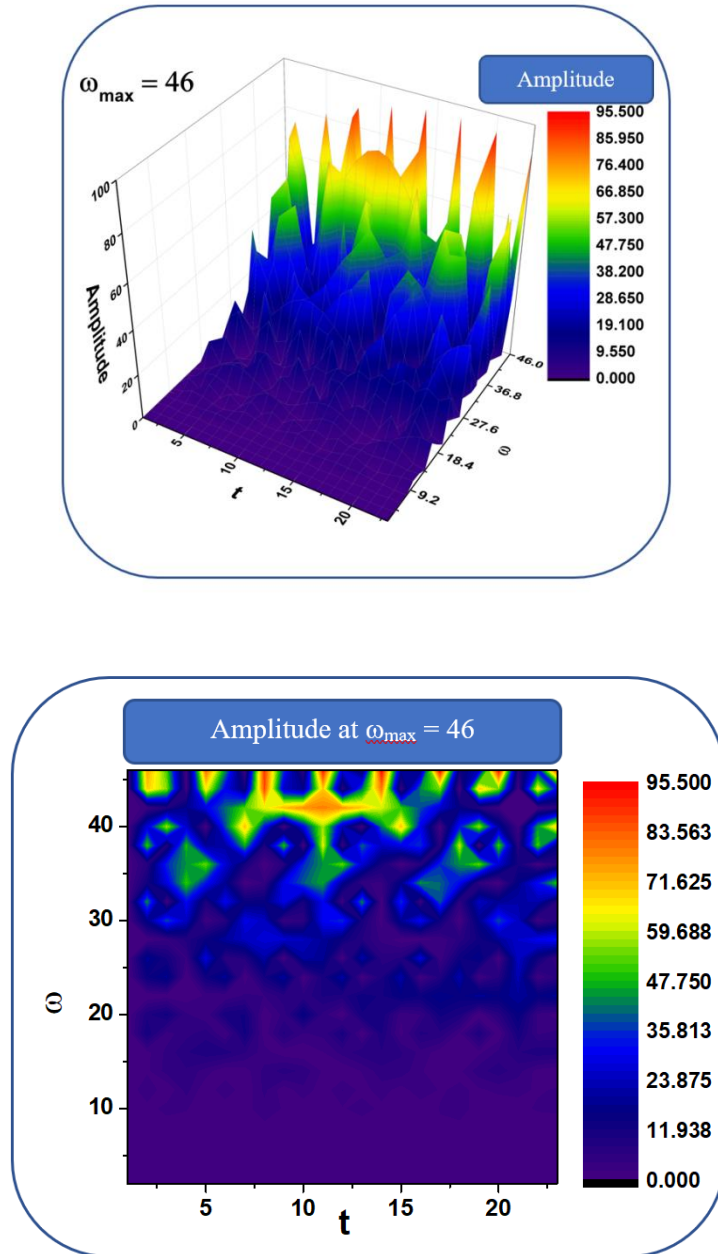values of the (pulsation–type) characteristic (10 and 15).

Fig. 2 – 3D and 2D representation of the solution Re(z) at various
scale resolution given by the maximum value of the (pulsation-type) characteristic
(27 and 46). Self-modulation of the signal can be observed.

3. Conclusions

The previous analysis specifies the fact that the temporal stochasticization meant to "transform" dissipative dynamics of a system into non-dissipative dynamics of the same system implies a Riccati-type "gauge" (the dynamic variables must satisfy a Riccati-type differential equation). The explicitation of such a situation can be found in a Stoler-type self-modulation (described by a Stoler transformation) of any signal. In other words, the temporal stochasticization processes of any bio-structure entities dynamics through Riccati-type gauge impose Stoler-type self-modulation of the signals which describe their dynamics.

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AUTOMODULAREA ÎN STRUCTURILE BIOLOGICE

(Rezumat)

Analizele de dinamică bazate pe ecuaţiile de propagare ale compuşilor electromagnetici biologici arată că conservarea elementului de suprafaţă în spaţiul fazelor implică statistici temporale speciale. Într-un asemenea cadru, identificarea statisticilor temporale menite de a transforma dinamici disipative în dinamici nedisipative implică etalonări Riccati la diverse rezoluţii de scară. O asemenea situaţie se realizează prin automodulare de tip Stoler în frecvenţă.